

$O(\alpha_s)$ corrections to the decays of polarized W^\pm and Z bosons into massive quark pairs

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Abstract

We present $O(\alpha_s)$ results on the decays of polarized W^\pm and Z bosons into massive quark pairs. The NLO QCD corrections to the polarized decay functions are given up to the second order in the quark mass expansion. We find a surprisingly strong dependence of the NLO polarized decay functions on finite quark mass effects even at the relatively large mass scale of the W^\pm and Z bosons. As a main application we consider the decay $t \rightarrow b + W^+$ involving the helicity fractions ρ_{mm} of the W^+ boson followed by the polarized decay $W^+(\uparrow) \rightarrow q_1 \bar{q}_2$ for which we determine the $O(\alpha_s)$ polar angle decay distribution. We also discuss NLO polarization effects in the production/decay process $e^+e^- \rightarrow Z(\uparrow) \rightarrow q\bar{q}$.

1 Introduction

The polarization of W^\pm and Z bosons produced in electroweak production processes is in general highly nontrivial. One therefore has a rich phenomenology of polarization effects in (W, Z) production and decay which will be explored in present and future experiments. The polarization of the W^\pm and Z bosons can be probed by decay correlations involving the decay products of the polarized (W, Z) bosons. A widely discussed prominent example of such decay correlations is the decay $t \rightarrow b + W^+$ followed by $W^+ \rightarrow \ell^+ \nu_\ell$ where the decay $W^+ \rightarrow \ell^+ \nu_\ell$ is used to analyze the helicity fractions of the W^+ resulting from the decay $t \rightarrow b + W^+$ (see e.g. Refs. [1, 2]). It would be interesting to explore the possibility to also make use of the quark–antiquark decay modes $W^\pm \rightarrow q_1 \bar{q}_2$ and $Z \rightarrow q \bar{q}$ to analyze the polarization of the (W^\pm, Z) bosons, in particular using the tagging modes $W^+ \rightarrow c \bar{b}, c \bar{s}$ and $Z \rightarrow c \bar{c}, b \bar{b}$ involving heavy quarks.

This paper is devoted to the calculation of NLO QCD effects in the decay of polarized (W, Z) gauge bosons into massless and massive quark–antiquark pairs. In order to provide a quick access to the importance of quark mass effects we have made use of a quark mass expansion of the rather lengthy fully analytic NLO results listed in [3]. We thereby demonstrate that, in polarized gauge boson decays involving charm and bottom quarks, the NLO finite mass effects are non-negligible. The reason is that the NLO finite mass corrections in polarized decays set in at linear order with rather large coefficients, contrary to the case of unpolarized decay where the finite mass effects set in only at $O(m_q^2/m_{W,Z}^2)$. Depending on the particular polarized decay function, the mass corrections can become as large as the leading term of the NLO mass expansion for decays involving b and c quarks.

NLO and finite mass effects will affect the decay correlations between the momenta of the production and decay process. As specific examples of how such correlations are affected by NLO and finite mass effects we consider the cascade decay process $t \rightarrow b + W^+(\uparrow)(\rightarrow c \bar{b})$ and the production/decay process $e^+ e^- \rightarrow Z(\uparrow) \rightarrow b \bar{b}, c \bar{c}$.

2 Angular decay distribution

The polar angle decay distribution of a polarized (W, Z) boson decaying into a fermion–antifermion pair (quark or lepton pair) is given by

$$\begin{aligned}
W(\theta) &= \sum_{m,m'=0,\pm 1} \rho_{mm} d_{mm'}^1(\theta) d_{mm'}^1(\theta) H_{m'm'} \\
&= \frac{3}{8}(1 + \cos^2 \theta) \left((\rho_{++} + \rho_{--}) (H_{++} + H_{--}) + 2\rho_{00}H_{00} \right) \\
&\quad + \frac{3}{4} \cos \theta \left((\rho_{++} - \rho_{--}) (H_{++} - H_{--}) \right) \\
&\quad + \frac{3}{4} \sin^2 \theta \left((\rho_{++} + \rho_{--})H_{00} + \rho_{00}(H_{++} + H_{--} - H_{00}) \right). \tag{1}
\end{aligned}$$

The diagonal spin density matrix elements of the polarized (W, Z) boson are denoted by ρ_{mm} .¹ They are defined in a (x, y, z) coordinate system associated with the production process while the polarized decay structure functions (for short: polarized decay functions) $H_{m'm'}$ are defined in a (x', y, z') coordinate system associated with the decay process. The two coordinate systems are rotated into each other by a rotation around the y axis by the polar angle θ . For example, for the sequential decay $t \rightarrow b + W^+(\uparrow)(\rightarrow c\bar{b})$ the z axis could be chosen to lie along the momentum of the W^+ in the top quark rest frame and the z' axis could be chosen to lie along the quark direction in the W^+ rest frame.

The polarized decay functions $H_{\pm\pm}$ and H_{00} stand for the probability of the decay of a polarized vector boson $(W, Z)(m')$ into a fermion–antifermion pair where the vector boson has the spin quantum numbers $m' = 0, \pm$ in the rotated (x', y, z') coordinate system. The spins of the fermion–antifermion pair are summed over when calculating the decay probability.

We take the spin density matrix to be normalized, i.e. $\rho_{++} + \rho_{00} + \rho_{--} = 1$. It is also convenient to define a normalized angular decay distribution $\widehat{W}(\theta) = W(\theta)/W$ where $W = \int_{-1}^1 W(\theta) d\cos\theta = \sum_m H_{mm}$ such that $\int_{-1}^1 \widehat{W}(\theta) d\cos\theta = 1$ (later on we denote

¹In the literature the diagonal elements of the normalized spin density matrix of the (W, Z) boson ρ_{mm} are frequently referred to as the helicity fractions \mathcal{F}_m of the gauge boson.

$W = \sum_m H_{mm}$ by H_{U+L}). Correspondingly we define normalized polarized decay functions $\hat{H}_{m'm'} = H_{m'm'} / \sum_m H_{mm}$.

The distribution Eq. (1) or its normalized form $\widehat{W}(\theta)$ is a second order equation in $\cos \theta$, i.e. it is the equation of a parabola with coefficients given by sums of the products $\rho_{mm} H_{m'm'}$. The parabola is upward bent for $(1 - 3\rho_{00})(1 - 3\hat{H}_{00}) > 0$ and downward bent for $(1 - 3\rho_{00})(1 - 3\hat{H}_{00}) < 0$. As a measure of the flatness of the decay distribution we define a convexity parameter c_f given by the differential change of slope (or the second derivative) of the decay distribution. From Eq. (1) one has

$$c_f = \frac{d^2 \widehat{W}}{d(\cos \theta)^2} = \frac{3}{4}(1 - 3\rho_{00})(1 - 3\hat{H}_{00}). \quad (2)$$

As a second global measure we introduce the forward-backward asymmetry of the decay distribution defined by

$$A_{FB} = \frac{W(F) - W(B)}{W(F) + W(B)} = \frac{3}{4}(\rho_{++} - \rho_{--})(\hat{H}_{++} - \hat{H}_{--}) \quad (3)$$

where $W(F) = W(0 \leq \theta \leq \pi/2)$ and $W(B) = W(\pi/2 \leq \theta \leq \pi)$.

Of interest is also the location of the extremum of the parabola in Eq. (1). The extremum is located at

$$\cos \theta \Big|_{\text{extr}} = - \frac{(\rho_{++} - \rho_{--})}{(1 - 3\rho_{00})} \frac{(\hat{H}_{++} - \hat{H}_{--})}{(1 - 3\hat{H}_{00})}. \quad (4)$$

The three measures are not independent for the normalized parabolic decay distributions $\widehat{W}(\theta)$. They are related by

$$\cos \theta \Big|_{\text{extr}} = - \frac{A_{FB}}{c_f}. \quad (5)$$

Note that all three measures factor into a production part described by the density matrix elements ρ_{mm} and a decay part given in terms of the polarized decay functions \hat{H}_{mm} .

As a well-known illustration consider the cascade decay $t \rightarrow b + W^+(\uparrow)$ followed by $W^+(\uparrow) \rightarrow f_1 \bar{f}_2$. At the Born term level and for massless fermions (quarks or leptons)

the only nonvanishing polarized decay function in the Standard Model (SM) is H_{++} if one chooses z' to lie along $\bar{f}_2 \in \{\ell^+, \bar{q}_2\}$. In the normalized form one has $\hat{H}_{++} = 1$. One obtains

$$\widehat{W}(\theta) = \frac{3}{8}(1 + \cos \theta)^2 \rho_{++} + \frac{3}{8}(1 - \cos \theta)^2 \rho_{--} + \frac{3}{4} \sin^2 \theta \rho_{00}. \quad (6)$$

The decay distribution (6) corresponds to a normalized parabola with convexity parameter $c_f = 3(1 - 3\rho_{00})/4$, a maximum at $\cos \theta|_{\text{extr}} = -(\rho_{++} - \rho_{--})/(1 - 3\rho_{00})$ and a forward-backward asymmetry of $A_{FB} = \frac{3}{4}(\rho_{++} - \rho_{--})$.

It is evident that an analysis of an angular decay distribution such as in Eq. (6) can be used to experimentally extract information on the helicity fractions ρ_{mm} ($m = \pm 1, 0$) of the W boson. Such an analysis has been widely applied using the leptonic decay modes of the W boson $W^+(\uparrow) \rightarrow \ell^+ \nu_\ell$ (see e.g. Refs. [1, 2]). Clearly the decay distribution (6) will be affected by radiative corrections and finite mass effects in as much as the three normalized decay structure functions \hat{H}_{mm} will change from the simple pattern $\hat{H}_{++} = 1$; $H_{00} = H_{--} = 0$ on which Eq. (6) is based. The modification of Eq. (6) in the quark sector through radiative corrections and finite mass effects is the subject of this paper. In this context it is important to note that for $t \rightarrow b + W^+(\uparrow)(\rightarrow q_1 \bar{q}_2)$ the radiative corrections to $t \rightarrow b + W^+(\uparrow)$ and $W^+(\uparrow) \rightarrow q_1 \bar{q}_2$ factorize at NLO in QCD (but not in higher orders), i.e. there is no NLO cross-talk between the production and decay processes [4].

Note that for unpolarized (W, Z) decay, when $\rho_{++} = \rho_{00} = \rho_{--} = 1/3$, Eq. (1) leads to a flat decay distribution

$$W(\theta) = \frac{1}{2}(H_{++} + H_{00} + H_{--}). \quad (7)$$

In the next two sections we shall first write out the diagonal spin density matrix elements ρ_{mm} of the (W, Z) boson in two prominent sample production processes and then, at a later stage, proceed to calculate the relevant set of decay structure functions H_{mm} at $O(\alpha_s)$. The results are then combined to present analytical and numerical results for the respective $O(\alpha_s)$ decay distributions.

3 Spin density matrix of the W boson

in the decay $t \rightarrow b + W^+(\uparrow)$

Let us start by discussing the spin density matrix of the W^+ in the decay $t \rightarrow b + W^+(\uparrow)$. As indicated by the notation, the W boson emerges in a polarized state in this decay.

The spin density matrix elements of the W^+ in $t \rightarrow b + W^+$ have been well studied. We take the z axis to lie along the momentum of the W^+ in the top quark rest frame. At LO one has [7]

$$\begin{aligned}\rho_{++}(Born) &= 0 && \rightarrow 0.0007, \\ \rho_{00}(Born) &= \frac{1}{1+2x^2} = 0.696 && \rightarrow 0.6887, \\ \rho_{--}(Born) &= \frac{2x^2}{1+2x^2} = 0.304 && \rightarrow 0.3106,\end{aligned}\tag{8}$$

where $x = m_W/m_t$. For the numerical values we use the central values of $m_W = 80.399 \pm 0.025$ GeV and $m_t = 172.0 \pm 0.9 \pm 1.3$ GeV provided by the Particle Data Group [6]. In Eq. (8) we have also given the NLO QCD results indicated by arrows (cf. Refs. [8, 9, 10, 11]).² The radiative correction to ρ_{++} can be seen to be very small. The absolute (relative) corrections to ρ_{00} and ρ_{--} amount to -0.73% (-1.05%) and 0.66% (2.17%).

The spin density matrix elements in Eq. (8) are calculated for unpolarized top decays. If the decaying top quark is polarized, the spin density matrix elements will depend on the orientation θ_P and the degree of polarization $P_t = |\vec{P}_t|$ of the top quark. The dependence is very simple for the $m_b = 0$ Born term case where one has

$$\rho_{++}^P = 0, \quad \rho_{00}^P = \rho_{00} \frac{1 + P_t \cos \theta_P}{D(\theta_P)}, \quad \rho_{--}^P = \rho_{--} \frac{1 - P_t \cos \theta_P}{D(\theta_P)},\tag{9}$$

and where the denominator in Eq. (9) is given by

$$D(\theta_P) = \rho_{00}(1 + P_t \cos \theta_P) + \rho_{--}(1 - P_t \cos \theta_P).\tag{10}$$

²The NNLO corrections to the spin density matrix elements of the W^+ have recently been calculated in Ref. [12].

It is clear that the relative weight of the two helicity fractions ρ_{00} and ρ_{--} can be changed by appropriately tuning the polarization of the top quark.

In the general case both helicity states of the polarized top quark are involved. This case is somewhat more complicated and has been worked out in Refs. [8, 9].

4 Spin density matrix of the Z boson

in the production process $e^+e^- \rightarrow Z(\uparrow)$

Next we discuss the polarization of the Z boson produced in e^+e^- annihilation where the polarization density matrix of the Z boson is purely transverse in the e^+e^- system. We take the z axis to lie along the e^- beam ($z \parallel e^-$). The Born term matrix element is given by (see e.g. Ref. [5])

$$\mathcal{M}(m) = -i\frac{g_Z}{4}\bar{v}(e^+)(v_e\gamma_\mu - a_e\gamma_\mu\gamma_5)u(e^-)\epsilon^{*\mu}(m) \quad (11)$$

($m = 0, \pm$) where, in the SM, one has

$$v_\ell = -1 + 4\sin^2\Theta_W \quad a_\ell = -1 \quad \text{for } \ell = e, \mu, \tau, \quad (12)$$

and where Θ_W is the Weinberg angle. In our numerical analysis we take $\sin^2\Theta_W = 0.231$ [6].

One can then work out the normalized diagonal spin density matrix elements of the Z boson in $e^+e^- \rightarrow Z(\uparrow)$ which are given by

$$\rho_{++} = \frac{(v_e - a_e)^2}{2(v_e^2 + a_e^2)} = 0.4244, \quad \rho_{00} = 0, \quad \rho_{--} = \frac{(v_e + a_e)^2}{2(v_e^2 + a_e^2)} = 0.5756. \quad (13)$$

Note that the two transverse density matrix elements in Eq. (13) are approximately equal due to the smallness of the vector current coupling constant $v_e = -0.076$.

Similar to the case $t \rightarrow b + W^+$, the spin density matrix elements of the Z boson can be tuned by polarizing the e^+e^- beams. Take, for example, longitudinally polarized beams

and denote the longitudinal polarization of the e^\mp beams by h^\mp , where the polarization is measured w.r.t. the momenta of the e^\mp beams. One then has

$$\rho_{++}^P = \rho_{++} \frac{(1+h^-)(1-h^+)}{D(h^-, h^+)}, \quad \rho_{00}^P = 0, \quad \rho_{--}^P = \rho_{--} \frac{(1-h^-)(1+h^+)}{D(h^-, h^+)}, \quad (14)$$

where

$$D(h^-, h^+) = \rho_{++}(1+h^-)(1-h^+) + \rho_{--}(1-h^-)(1+h^+). \quad (15)$$

For example, with a 100% longitudinally polarized electron beam one obtains $\rho_{--}^P = 1$ for $h^- = -1$ and $\rho_{++}^P = 1$ for $h^- = +1$.

5 Polarized W^\pm decays into massive quark pairs

We first treat the case $W^+ \rightarrow q_1 \bar{q}_2$. The LO Born term amplitude is given by (see e.g. [5])

$$\mathcal{M}_W(m) = -i \frac{g_W}{\sqrt{2}} V_{q_1 q_2} \bar{u}_1(p_1) \gamma^\mu \frac{1-\gamma_5}{2} v_2(p_2) \epsilon_\mu^W(m), \quad (16)$$

where $g_W = e/\sin \Theta_W$ is the electroweak coupling constant and the $V_{q_1 q_2}$ are Kobayashi–Maskawa matrix elements. Let us define a reduced matrix element $\widetilde{\mathcal{M}}(m)$ by splitting off the coupling factors and the factor $1/2$ from the chiral projector such that

$$\widetilde{\mathcal{M}}_W(m) = \bar{u}_1(p_1) \gamma^\mu (1-\gamma_5) v_2(p_2) \epsilon_\mu^W(m). \quad (17)$$

The LO polarized decay functions H_{mm} are then obtained from

$$H_{mm} = N_c \sum_{\text{quark spins}} \widetilde{\mathcal{M}}_W(m) \widetilde{\mathcal{M}}_W^\dagger(m). \quad (18)$$

Our aim is to calculate the NLO polarized decay functions H_{mm} of a W^+ boson decaying into a heavy quark pair where the W^+ has definite spin quantum numbers $m = \pm, 0$. We first discuss a coordinate system where the z axis lies along the quark momentum ($z \parallel q_1$) (system I). The Born term and the α_s contributions to the polarized decay functions $H_{\pm\pm}$ and H_{00} and the unpolarized total decay function $H_{U+L} = H_{++} + H_{00} + H_{--}$ are expanded

up to the second order in the quark mass ratios $\sqrt{\mu_{1,2}} = m_{1,2}/m_W$ where we make use of the unexpanded analytical results given in Ref. [3]. One obtains

$$H_{++}^I = 8N_c q^2 \left[0 + \dots + \frac{\alpha_s}{6\pi} \left(1 + (\pi^2 - 16)\sqrt{\mu_1} + (5 + 2\pi^2/3)\mu_1 + \mu_2 - 2\mu_1 \ln \mu_1 + \dots \right) \right], \quad (19)$$

$$H_{00}^I = 8N_c q^2 \left[0 + \frac{(\mu_1 + \mu_2)}{2} + \dots + \frac{\alpha_s}{6\pi} \left(4 - 2\pi^2\sqrt{\mu_1} - (39 + 8\pi^2/3)\mu_1 + \mu_2 - 30\mu_1 \ln \mu_1 + 6\mu_2 \ln \mu_2 - 2\mu_1 \ln^2 \mu_1 + \dots \right) \right], \quad (20)$$

$$H_{--}^I = 8N_c q^2 \left[1 - \mu_1 - \mu_2 + \dots + \frac{\alpha_s}{6\pi} \left(1 + (\pi^2 + 16)\sqrt{\mu_1} + (49 + 2\pi^2)\mu_1 + 13\mu_2 + 14\mu_1 \ln \mu_1 - 24\mu_2 \ln \mu_2 + 2\mu_1 \ln^2 \mu_1 + \dots \right) \right]. \quad (21)$$

We truncate the mass expansion at $O(\mu_i)$ since third order quark mass effects can be expected to be quite small judging from the fact that $\mu_c^{3/2} = (1.5/80.399)^3 = 6.49 \cdot 10^{-6}$ and $\mu_b^{3/2} = (4.8/80.399)^3 = 0.21 \cdot 10^{-3}$.

For the sum of the three polarized decay functions $\sum_m H_{mm} := H_{U+L}$ one has

$$\begin{aligned} H_{U+L} &= H_{U+L}^I = H_{++}^I + H_{00}^I + H_{--}^I \\ &= 8N_c q^2 \left[1 - \frac{(\mu_1 + \mu_2)}{2} + \dots + \frac{\alpha_s}{6\pi} \left(6 + 15\mu_1 + 15\mu_2 - 18\mu_1 \ln \mu_1 - 18\mu_2 \ln \mu_2 + \dots \right) \right]. \end{aligned} \quad (22)$$

Note that the sum of the polarized decay functions is independent of the choice of the z axis as is indicated in Eq. (22).

The mass corrections set in quadratically in the LO polarized decay functions and also in the radiatively corrected unpolarized decay function H_{U+L} . In contrast to this, the mass corrections to the radiatively corrected polarized decay functions $H_{\pm\pm}$ and H_{00} set in linearly. Surprisingly, some of the linear mass corrections carry rather large coefficients such as the coefficient $(\pi^2 + 16) = 25.87$ multiplying the linear mass term $\sqrt{\mu_1}$ in H_{--}^I in

Eq. (21). Contrary to naive expectations one therefore needs to keep finite mass effects in the radiatively corrected polarized decay functions for massive quark pair production even at the relatively large mass scale of the W mass. Note that the radiatively corrected unpolarized decay function H_{U+L} is symmetric in the quark masses whereas the polarized decay functions show a large quark mass asymmetry at NLO.

In order to make contact with the unpolarized decay rate $\Gamma(W^+ \rightarrow q_1 \bar{q}_2)$, we define polarized decay rates

$$\Gamma_{mm} = \frac{|\vec{p}|}{64\pi m_W^2} g_W^2 |V_{q_1 q_2}|^2 H_{mm} \quad (23)$$

($|\vec{p}| = \frac{m_W}{2}(1 + \mu_1^2 + \mu_2^2 - 2\mu_1 - 2\mu_2 - 2\mu_1\mu_2)^{1/2}$) which, for the unpolarized rate, gives

$$\Gamma(W^+ \rightarrow q_1 \bar{q}_2) = \frac{1}{3}(\Gamma_{++} + \Gamma_{00} + \Gamma_{--}) = \frac{|\vec{p}|}{192\pi m_W^2} g_W^2 |V_{q_1 q_2}|^2 H_{U+L}. \quad (24)$$

The quark mass corrections will be most important for the decay $W^+ \rightarrow c\bar{b}$. For the quark masses we take $m_c = 1.5 \text{ GeV}$ and $m_b = 4.8 \text{ GeV}$. For the $W^+ \rightarrow c\bar{b}$ unpolarized decay function H_{U+L} one obtains

$$H_{U+L} = 8N_c q^2 \left[0.998 + \dots + \frac{\alpha_s}{6\pi} (6 + 0.470 + \dots) \right]. \quad (25)$$

In order to highlight the numerical importance of quark mass effects we have separately listed the leading and the $O(\mu_i)$ quark mass effects in the NLO terms in Eq. (25). Even though the mass corrections to the sum of the polarized decay functions set in only quadratically, the mass effects in the NLO corrections can be seen to amount to a non-negligible $O(8\%)$ where the largest contribution comes from the bottom quark term $-18\mu_2 \ln \mu_2 = 0.0071$. Using $\alpha_s(m_W^2) = 0.117$ one finds an overall increase of the zero mass Born term decay rate by 3.8%, i.e. one has

$$\Gamma(\text{NLO}; O(\mu_i)) = 1.038 \cdot \Gamma(\text{Born}; \mu_i = 0), \quad (26)$$

where the bulk of the increase comes from the NLO zero mass term.

For the $W^+ \rightarrow c\bar{b}$ polarized decay functions one obtains

$$\begin{aligned} H_{++}^I &= 8N_c q^2 [0 + \dots + \frac{\alpha_s}{6\pi} (1 - 0.101 + \dots)], \\ H_{00}^I &= 8N_c q^2 [0.002 + \dots + \frac{\alpha_s}{6\pi} (4 - 0.469 + \dots)], \\ H_{--}^I &= 8N_c q^2 [0.996 + \dots + \frac{\alpha_s}{6\pi} (1 + 1.040 + \dots)]. \end{aligned} \quad (27)$$

The mass corrections to the NLO polarized decay functions can be seen to be large. The largest mass correction occurs for H_{--}^I which is the only polarized decay function which is nonzero at the $m_q = 0$ Born term level. The mass correction to the leading NLO contribution in H_{--}^I is of $O(100\%)$.

The large NLO mass correction to H_{--}^I does not, however, feed through to the normalized decay distribution which is governed by the normalized polarized decay functions $\hat{H}_{mm}^I = H_{mm}^I / H_{U+L}$. This can be appreciated by writing out the polarized decay functions in a generic notation where the (small) mass corrections to the Born term contributions are neglected. One has

$$\begin{aligned} H_{++} &= 8N_c q^2 (0 + \frac{\alpha_s}{6\pi} (1 + \mu_{++})) \\ H_{00} &= 8N_c q^2 (0 + \frac{\alpha_s}{6\pi} (4 + \mu_{00})) \\ H_{--} &= 8N_c q^2 (1 + \frac{\alpha_s}{6\pi} (1 + \mu_{--})) \end{aligned} \quad (28)$$

where the μ_{mm} denote the respective NLO finite mass corrections. One then expands the normalized polarized decay functions \hat{H}_{mm} in the strong coupling constant α_s . In this approximation one has

$$\begin{aligned} \hat{H}_{++} &= 0 + \frac{\alpha_s}{6\pi} (1 + \mu_{++}) \\ \hat{H}_{00} &= 0 + \frac{\alpha_s}{6\pi} (4 + \mu_{00}) \\ \hat{H}_{--} &= 1 + \frac{\alpha_s}{6\pi} (-5 - \mu_{++} - \mu_{00}). \end{aligned} \quad (29)$$

It is apparent that the NLO finite mass corrections to the normalized polarized decay functions \hat{H}_{mm} are solely determined by the $O(10\%)$ mass corrections μ_{++} and μ_{00} . Since

μ_{++} and μ_{00} are negative, the NLO mass corrections are destructive. Numerically the NLO mass corrections amount to only $O(-10\%)$ of the leading NLO mass term (see Eq. (27)). The $O(100\%)$ NLO mass correction to H_{--} drops out when normalizing the polarized decay functions.

We are now in the position to write down numerical results for the angular decay distribution (1). We take into account $O(\alpha_s)$ results both for the density matrix elements ρ_{mm} and the decay structure functions H_{mm} . At $O(\alpha_s)$ one thus has to take the sum $\rho_{mm}(\text{Born})H_{m'm'}(\alpha_s) + \rho_{mm}(\alpha_s)H_{m'm'}(\text{Born})$. As mentioned before, there is no $O(\alpha_s)$ cross-talk between the production and the decay process because the intermediate gauge boson is colour neutral [4]. As before we concentrate on the decay $W^+ \rightarrow c\bar{b}$. For system I one finds $(\alpha_s(m_W^2) = 0.117)$

$$\widehat{W}^{\text{I}}(\theta) = \frac{3}{8}(1 + \cos \theta)^2 \begin{Bmatrix} 0.305 \\ 0.318 \\ 0.318 \end{Bmatrix} + \frac{3}{8}(1 - \cos \theta)^2 \begin{Bmatrix} 0.001 \\ 0.019 \\ 0.018 \end{Bmatrix} + \frac{3}{4} \sin^2 \theta \begin{Bmatrix} 0.694 \\ 0.663 \\ 0.664 \end{Bmatrix} \quad (30)$$

For the sake of comparison we have listed three numerical values each for the angular coefficients. The top entry is for (Born; $\mu_i \neq 0$), the middle entry is for ($O(\alpha_s)$; $\mu_i = 0$), and the bottom entry is for ($O(\alpha_s)$; $\mu_i \neq 0$). The same three-tiered notation will be used in subsequent formulas and in Table 1.

Eq. (30) describes a downward bent parabola with unit area and, as Fig. 1 shows, a maximum slightly displaced to the right of $\cos \theta = 0$. By comparing the respective numbers in the normalized decay distribution $\widehat{W}(\theta)$ in Eq. (30), NLO quark mass effects can be seen to be almost negligible even if they are important for the polarized decay functions. More important are the radiative corrections which lead to a 2.0% enhancement (on an absolute scale) of the normalized decay distribution at the forward point ($\cos \theta = 1$) and a 2.7% enhancement at the backward point ($\cos \theta = -1$). Midways at $\cos \theta = 0$ one finds a 1.2% depletion of the decay distribution. This is illustrated in Fig. 1 where we compare the $O(\mu_i)$ normalized angular decay distributions for the Born term case and the $O(\alpha_s)$ case. The net effect of the radiative corrections is to make the normalized angular decay distribution

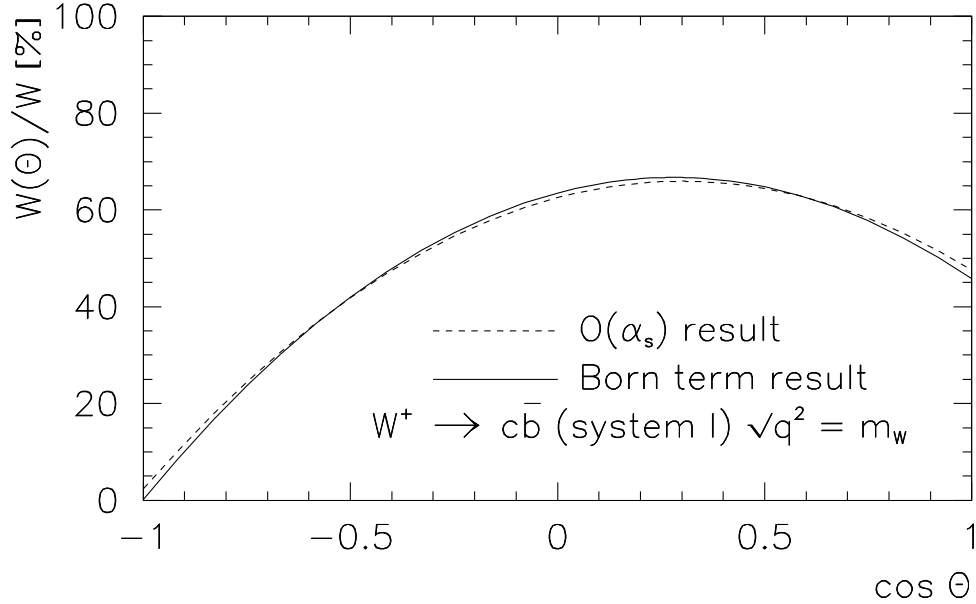


Figure 1: Normalized angular decay distribution $\widehat{W}(\theta) = W(\theta)/W$ for $W^+ \rightarrow c\bar{b}$ at LO (full line) and NLO (dotted line) in system I including $O(m_{c,b}^2)$ finite quark mass contributions.

flatter with little dependence on quark mass effects.

In Table 1 we present our numerical results for the three global parameters c_f , A_{FB} and $\cos\theta|_{\text{extr}}$ that characterize the decay distribution. In Table 1 we use the same three-tiered notation as in Eq. (30). The numerical results for system I ($z \parallel c$) are listed in column 1. We emphasize that the inverse of the denominators factors occurring in the calculation of the global measures in Eqs. (2–4) have been left unexpanded in α_s when calculating the entries in Table 1. The initial and final state QCD corrections can be seen to reduce the convexity parameter c_f and the forward-backward asymmetry by 8.37 % and 1.25 %, respectively. The maximum of the decay distribution is shifted to the right by 7.7 %. By comparing the numbers in tier 3 with those in tier 2 one can see that the bulk of these shifts come from the NLO zero mass corrections. NLO finite mass effects are small and tend to slightly reduce the NLO zero mass corrections.

The largest NLO corrections come from the final state corrections. One can therefore obtain a rough understanding of the numbers in Table 1 by neglecting the initial state

corrections and the (small) finite mass effects in the Born term contributions. Expanding the relevant contributions to $O(\alpha_s)$ one obtains

$$c_f^I = \frac{3}{4}(1 - 3\rho_{00})\left(1 - \frac{\alpha_s}{6\pi}(12 - 1.41)\right) = -0.762, \quad (31)$$

$$A_{FB}^I = -\frac{3}{4}(\rho_{++} - \rho_{--})\left(1 - \frac{\alpha_s}{6\pi}(6 - 0.671)\right) = 0.221, \quad (32)$$

$$\cos\theta\Big|_{\max}^I = \frac{\rho_{++} - \rho_{--}}{1 - 3\rho_{00}}\left(1 + \frac{\alpha_s}{6\pi}(6 - 0.736)\right) = 0.289. \quad (33)$$

Looking at the different contributions in Eq. (31–33) one can understand the main features of the numerical results listed in Table 1 that were already discussed above.

In Eqs. (31–33) we also quote approximate numbers which are calculated from the finite mass corrections μ_{++} and μ_{--} listed in Eq. (27) and the Born term values for the spin density elements ρ_{mm} listed in Eq. (8). The approximate numbers listed in Eqs. (31–33) can be seen to deviate by small amounts from the exact numbers listed in Table 1 where the largest deviation occurs for A_{FB}^I and $\cos\theta\Big|_{\max}^I$.

Summarizing our results in system I one finds that NLO and NLO quark mass effects are important for the polarized decay functions. The NLO quark mass effects, however, do not feed through to the angular decay distribution which is mainly affected by the leading mass term in the final state radiative correction. One concludes that quark mass effects are quite small for the radiatively corrected normalized angular decay distribution even if they are important for the radiatively corrected decay functions.

When the polar angle θ is measured w.r.t. the antiquark direction ($z \parallel \bar{q}$) (system II), the relevant expressions for the helicity structure functions can be obtained from those in system I by the exchange $\mu_1 \leftrightarrow \mu_2$, and $H_{\pm\pm} \leftrightarrow H_{\mp\mp}$ and $H_{00} \leftrightarrow H_{00}$. One has

$$H_{\mp\mp}^{\text{II}}(\mu_1, \mu_2) = H_{\pm\pm}^{\text{I}}(\mu_2, \mu_1), \quad H_{00}^{\text{II}}(\mu_1, \mu_2) = H_{00}^{\text{I}}(\mu_2, \mu_1). \quad (34)$$

Because of the mass asymmetry of the polarized decay functions and because one is exchanging $m_c \leftrightarrow m_b$, the quark mass effects are more pronounced in system II. In fact, one

	$W^+ \rightarrow c\bar{b}$		$Z \rightarrow c\bar{c}$	$Z \rightarrow b\bar{b}$
	system I ($z \parallel c$)	system II ($z \parallel \bar{b}$)	system I ($z \parallel c$)	system I ($z \parallel b$)
c_f	-0.811 -0.741 -0.743	-0.811 -0.741 -0.752	+0.750 +0.697 +0.701	+0.746 +0.697 +0.706
A_{FB}	+0.228 +0.224 +0.225	-0.228 -0.224 -0.226	+0.076 +0.073 +0.073	+0.106 +0.102 +0.103
$\cos \theta \big _{\text{extr}}$	+0.281 +0.304 +0.302	-0.281 -0.304 -0.301	-0.101 -0.105 -0.105	-0.142 -0.147 -0.146

Table 1: The parameters c_f , A_{FB} and $\cos \theta \big|_{\text{extr}}$ characterizing the normalized polar angle decay distribution of the cascade decays $t \rightarrow b + W^+(\uparrow)(\rightarrow c\bar{b})$ and the production/decay process $e^+e^- \rightarrow Z(\uparrow) \rightarrow b\bar{b}, c\bar{c}$. We use a three-tiered notation where the top entries are for (Born; $\mu_i \neq 0$), the middle entry is for ($O(\alpha_s)$; $\mu_i = 0$), and the bottom entry is for ($O(\alpha_s)$; $\mu_i \neq 0$).

now has

$$\begin{aligned}
H_{++}^{\text{II}} &= 8N_c q^2 [0.996 + \dots + \frac{\alpha_s}{6\pi} (1 + 1.806 + \dots)], \\
H_{00}^{\text{II}} &= 8N_c q^2 [0.002 + \dots + \frac{\alpha_s}{6\pi} (4 - 1.051 + \dots)], \\
H_{--}^{\text{II}} &= 8N_c q^2 [0 + \dots + \frac{\alpha_s}{6\pi} (1 - 0.284 + \dots)].
\end{aligned} \tag{35}$$

The NLO quark mass corrections in system II can be seen to be approximately two-and-a-half times larger than those in system I. The largest mass correction now occurs for H_{++}^{II} .

Numerically one obtains

$$\widehat{W}^{\text{II}}(\theta) = \frac{3}{8}(1 + \cos \theta)^2 \left\{ \begin{array}{c} 0.001 \\ 0.019 \\ 0.016 \end{array} \right\} + \frac{3}{8}(1 - \cos \theta)^2 \left\{ \begin{array}{c} 0.305 \\ 0.318 \\ 0.317 \end{array} \right\} + \frac{3}{4} \sin^2 \theta \left\{ \begin{array}{c} 0.694 \\ 0.663 \\ 0.667 \end{array} \right\}. \tag{36}$$

Again one observes that the large quark mass effects seen in the polarized decay functions do not feed through to the angular decay distribution. Of course, the reason is the same

as explained after Eq. (29). Except for the slightly enhanced NLO quark mass effects, the distribution (36) is just a reflection of Eq. (30) at the line $\cos\theta = 0$, i.e. $A_{FB}^I \sim -A_{FB}^{II}$.

Larger quark mass effects can also be seen in the convexity parameter c_f^{II} , in the forward-backward asymmetry A_{FB}^{II} and in the position of the maximum $\cos\theta|_{\max}^{II}$ (see Table 1). The larger size of the NLO finite mass corrections to the global measures can be inferred by listing approximate formulas similar to those in Eqs. (31–33). One has

$$c_f^{II} = \frac{3}{4}(1 - 3\rho_{00})\left(1 - \frac{\alpha_s}{6\pi}(12 - 3.153)\right) = -0.771, \quad (37)$$

$$A_{FB}^{II} = -\frac{3}{4}(\rho_{++} - \rho_{--})\left(1 - \frac{\alpha_s}{6\pi}(6 - 1.619)\right) = 0.222, \quad (38)$$

$$\cos\theta|_{\max}^{II} = \frac{\rho_{++} - \rho_{--}}{1 - 3\rho_{00}}\left(1 + \frac{\alpha_s}{6\pi}(6 - 1.534)\right) = 0.287. \quad (39)$$

One can see that the NLO finite mass effects make up $\sim 25\%$ of the leading NLO terms as compared to the $\sim 10\%$ in system I (see Eqs. (31–33)). Again the approximate numbers listed in Eqs. (37–39) can be seen to only deviate by small amounts from the exact numbers listed in Table 1 (column 2; tier 3). Again one concludes that, even though quark mass effects are larger in system II, the bulk of the radiative corrections still come from the NLO leading terms $\propto 12\alpha_s/(6\pi)$ and $\propto 6\alpha_s/(6\pi)$.

Up to this point we have only considered the decay $W^+ \rightarrow q_1 \bar{q}_2$. The charge conjugated decay $W^- \rightarrow \bar{q}_1 q_2$ is related to $W^+ \rightarrow q_1 \bar{q}_2$ by CP -invariance. The corresponding helicity structure functions $H_{mm}(W^- \rightarrow \bar{q}_1 q_2)$ can be obtained via the relations

$$H_{mm}(W^- \rightarrow \bar{q}_1 q_2; \mu_1, \mu_2; z \parallel q_2) = H_{mm}(W^+ \rightarrow q_1 \bar{q}_2; \mu_2, \mu_1; z \parallel q_1), \quad (40)$$

where the z axis for the decay $W^- \rightarrow \bar{q}_1 q_2$ lies along the quark direction ($z \parallel q_2$).

6 Polarized Z decays into massive quark pairs

In the SM the Born term matrix element for the decay $Z(m) \rightarrow q\bar{q}$ with spin quantum numbers m is given by (see e.g. Ref. [5])

$$\mathcal{M}_Z(m) = -i \frac{g_Z}{4} \bar{u}(q) (v_f \gamma_\mu - a_f \gamma_\mu \gamma_5) v(\bar{q}) \epsilon^\mu(m) \quad (41)$$

where $g_Z^2 = 8G_F M_Z^2 / \sqrt{2}$, and where, in the SM, one has

$$v_f = 1 - \frac{8}{3} \sin^2 \Theta_W, \quad a_f = 1 \quad \text{for } u, c, t, \quad (42)$$

$$v_f = -1 + \frac{4}{3} \sin^2 \Theta_W, \quad a_f = -1 \quad \text{for } d, s, b. \quad (43)$$

Similar to Eq. (17) we define reduced amplitudes by writing

$$\widetilde{\mathcal{M}}_Z(m) = \bar{u}(q) (v_f \gamma_\mu - a_f \gamma_\mu \gamma_5) v(\bar{q}) \epsilon^\mu(m). \quad (44)$$

As in Eq. (18), the LO polarized decay functions H_{mm} are calculated according to

$$H_{mm} = N_c \sum_{\text{quark spins}} \widetilde{\mathcal{M}}_Z(m) \widetilde{\mathcal{M}}_Z^\dagger(m). \quad (45)$$

Compared to the charged current case the relative weights of the vector (V) and axial vector current (A) contributions are no longer simple and it is more convenient to switch to a notation in terms of the VV , AA and $VA = AV$ contributions. Again we make use of the analytical results in Ref. [3] (or those in Ref. [13]) which we expand up to $O(\mu)$. One obtains

$$\begin{aligned} H_U^{VV/AA} &= 4N_c q^2 \left[1 - 2\mu \pm 2\mu \dots + \frac{\alpha_s}{6\pi} \left(2 + 2\pi^2 \sqrt{\mu} + \left(68 + \frac{8\pi^2}{3} \right) \mu \right. \right. \\ &\quad \left. \left. - 12\mu \ln \mu + 2\mu \ln^2 \mu \pm 24\mu (1 + \ln \mu) \dots \right) \right], \\ H_F^{VA/AV} &= 4N_c q^2 \left[1 - 2\mu \dots + \frac{\alpha_s}{6\pi} \left(32\sqrt{\mu} + \left(56 + \frac{4\pi^2}{3} \right) \mu - 8\mu \ln \mu \right. \right. \\ &\quad \left. \left. + 2\mu \ln^2 \mu \dots \right) \right], \\ H_L^{VV/AA} &= 4N_c q^2 \left[\mu \pm \mu \dots + \frac{\alpha_s}{6\pi} \left(4 - 2\pi^2 \sqrt{\mu} - \left(38 + \frac{8\pi^2}{3} \right) \mu \right. \right. \\ &\quad \left. \left. - 24\mu \ln \mu - 2\mu \ln^2 \mu \pm 6\mu (5 + 2 \ln \mu) \dots \right) \right]. \end{aligned} \quad (46)$$

Note that, in the zero mass limit, there are no α_s corrections to the parity-violating structure function H_F^{VA} , as noted before in Ref. [14, 15].

As in system I of the charged current case we evaluate the polarized decay functions $H_{\pm\pm}$ and H_{00} in a system where the quark lies along the z direction ($z \parallel q$). One has

$$\begin{aligned} H_{\pm\pm}^I &= \frac{1}{2} \left(v_f^2 H_U^{VV} + a_f^2 H_U^{AA} \mp 2v_f a_f H_F^{VA} \right), \\ H_{00}^I &= v_f^2 H_L^{VV} + a_f^2 H_L^{AA}. \end{aligned} \quad (47)$$

As a check on Eq. (47) one can set $v_f = a_f = 1$ or $v_f = a_f = -1$ and one will then recover the charged current results Eq. (19) (system I) with $\mu_1 = \mu_2 = \mu$. When the z axis is taken to be along the antiquark direction (system II; $z \parallel \bar{q}$), there will be no change for H_{00} but one needs to exchange $H_{\pm\pm} \leftrightarrow H_{\mp\mp}$ (or, $H_F^{VA} \leftrightarrow -H_F^{VA}$) in Eq. (47).

For the sum of the three polarized decay functions one obtains

$$\begin{aligned} H_{U+L} &= H_{U+L}^I = H_{++}^I + H_{00}^I + H_{--}^I = v_f^2 H_{U+L}^{VV} + a_f^2 H_{U+L}^{AA} \\ &= 8N_c q^2 \left\{ v_f^2 \left[\frac{1}{2} + \mu \dots + \frac{\alpha_s}{6\pi} (3 + 42\mu + \dots) \right] \right. \\ &\quad \left. + a_f^2 \left[\frac{1}{2} - 2\mu \dots + \frac{\alpha_s}{6\pi} (3 - 12\mu - 36\mu \ln \mu + \dots) \right] \right\}. \end{aligned} \quad (48)$$

In order to make contact with the total decay rate $\Gamma(Z \rightarrow q\bar{q})$, we define polarized decay rates by

$$\Gamma_{mm} = \frac{G_F |\vec{p}|}{16\pi\sqrt{2}} H_{mm} \quad (49)$$

where $|\vec{p}| = \frac{m_Z}{2} \sqrt{1 - 4\mu}$. For the unpolarized decay rate one then obtains

$$\Gamma(Z \rightarrow q\bar{q}) = \frac{1}{3} (\Gamma_{++} + \Gamma_{00} + \Gamma_{--}) = \frac{G_F |\vec{p}|}{48\pi\sqrt{2}} H_{U+L}. \quad (50)$$

Returning to Eq. (46), we write down our numerical results for the decay $Z \rightarrow b\bar{b}$. In system I one has

$$\begin{aligned} H_U^{VV}(b\bar{b}) &= 4N_c q^2 \left[1 + \frac{\alpha_s}{6\pi} (2 + 1.363 + \dots) \right], \\ H_U^{AA}(b\bar{b}) &= 4N_c q^2 \left[0.989 + \frac{\alpha_s}{6\pi} (2 + 2.013 + \dots) \right], \end{aligned}$$

$$\begin{aligned}
H_F^{VA}(b\bar{b}) &= 4N_c q^2 \left[0.994 + \frac{\alpha_s}{6\pi} (0 + 2.199 + \dots) \right], \\
H_L^{VV}(b\bar{b}) &= 4N_c q^2 \left[0.006 + \frac{\alpha_s}{6\pi} (4 - 1.131 + \dots) \right], \\
H_L^{AA}(b\bar{b}) &= 4N_c q^2 \left[0 + \frac{\alpha_s}{6\pi} (4 - 0.905 + \dots) \right],
\end{aligned} \tag{51}$$

or, using Eq. (47)

$$\begin{aligned}
H_{++}^I(b\bar{b}) &= 4N_c q^2 \left[0.046 + \frac{\alpha_s}{6\pi} (1.479 - 0.189 + \dots) \right], \\
H_{00}^I(b\bar{b}) &= 4N_c q^2 \left[0.003 + \frac{\alpha_s}{6\pi} (5.916 - 1.447 + \dots) \right], \\
H_{--}^I(b\bar{b}) &= 4N_c q^2 \left[1.422 + \frac{\alpha_s}{6\pi} (1.479 + 2.855 + \dots) \right].
\end{aligned} \tag{52}$$

The NLO quark mass effects in the polarized decay functions can be seen to be large. The largest NLO quark mass correction arises in the polarized decay function $H_{--}^I(b\bar{b})$ where the mass correction amounts to a $O(200\%)$ effect compared to the leading NLO term.

For the unpolarized decay function H_{U+L} one obtains

$$H_{U+L}(b\bar{b}) = 4N_c q^2 \left[1.471 + \frac{\alpha_s}{6\pi} (8.874 + 1.219 + \dots) \right] \tag{53}$$

The $O(\mu_b)$ quark mass and radiative corrections effects increase the zero mass Born term decay rate by 3.7%, i.e. one has (we take $\alpha_s(m_Z^2) = 0.115$)

$$\Gamma(\text{NLO}; Z \rightarrow b\bar{b}, O(\mu_b)) = 1.037 \cdot \Gamma(\text{Born}; Z \rightarrow b\bar{b}, \mu_b = 0). \tag{54}$$

Next we write down the angular decay distribution for $Z(\uparrow) \rightarrow q\bar{q}$ with Z polarization obtained from the production process $e^+e^- \rightarrow Z(\uparrow)$. In terms of the VV , AA and VA structure functions in Eq. (46) one has

$$\begin{aligned}
W(\theta) &= \frac{3}{8} (1 + \cos^2 \theta) \left(v_f^2 H_U^{VV} + a_f^2 H_U^{AA} \right) \\
&\quad + \frac{3}{4} \cos \theta \frac{2v_e a_e 2v_f a_f}{v_e^2 + a_e^2} H_F^{VA} \\
&\quad + \frac{3}{4} \sin^2 \theta \left(v_f^2 H_L^{VV} + a_f^2 H_L^{AA} \right).
\end{aligned} \tag{55}$$

Note that the electroweak parameters v_e and a_e do not appear in the first and last row of Eq. (55) because we are using normalized density matrix elements such that $\rho_{++} + \rho_{--} = 1$.

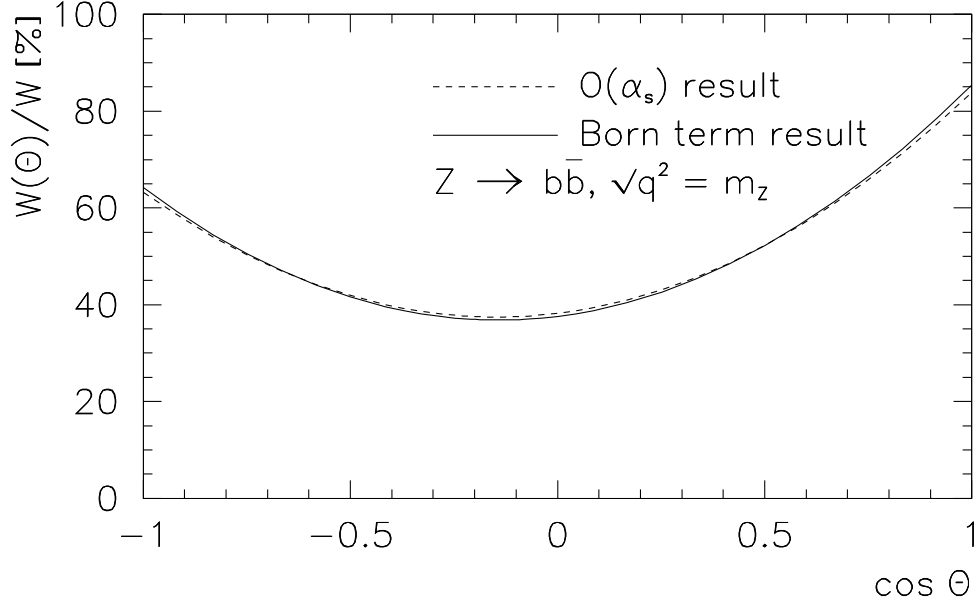


Figure 2: Normalized angular decay distribution $\widehat{W}(\theta) = W(\theta)/W$ for $Z \rightarrow b\bar{b}$ at LO (full line) and NLO (dotted line), including $O(m_c^2)$ finite quark mass contributions.

The normalized decay distribution is obtained from Eq. (55) through $\widehat{W}(\theta) = W(\theta)/W$ where $W = (v_f^2 H_{U+L}^{VV} + a_f^2 H_{U+L}^{AA})$.

Numerically one obtains ($m_Z = 91.188 \text{ GeV}$, $\alpha_s(m_Z^2) = 0.115$)

$$\widehat{W}^I(\theta)(b\bar{b}) = \frac{3}{8}(1 + \cos \theta)^2 \begin{Bmatrix} 0.570 \\ 0.556 \\ 0.559 \end{Bmatrix} + \frac{3}{8}(1 - \cos \theta)^2 \begin{Bmatrix} 0.428 \\ 0.420 \\ 0.421 \end{Bmatrix} + \frac{3}{4} \sin^2 \theta \begin{Bmatrix} 0.002 \\ 0.024 \\ 0.020 \end{Bmatrix}. \quad (56)$$

Eq. (56) describes an upward bent parabola with unit area and a minimum slightly displaced to the left of $\cos \theta = 0$ (see Fig. 2). Final state radiative corrections have a $O((1-2)\%)$ effect (on an absolute scale) on the coefficient functions of the angular decay distribution while NLO quark mass effects contribute only at the per mill level. In Fig. 2 we compare the $O(\mu)$ angular decay distributions for the Born term case and the $O(\alpha_s)$ case. The NLO corrections can be seen to make the angular decay distribution flatter. In fact, the convexity parameter is reduced by 5.36% through the radiative corrections as Table 1 shows. The decay distribution is weighted towards the forward hemisphere such that A_{FB}

is positive. It is barely visible that the radiative corrections reduce the forward-backward asymmetry. This is born out in Table 1 where one finds a 2.73% reduction in A_{FB} . The minimum is slightly shifted to the left. Quantitatively, this amounts to a 2.78% effect as Table 1 shows.

A rough description of the global effects of the radiative corrections on the decay distribution can again be obtained by approximate formulas using the same set of approximations as in Eq. (31-33). One now obtains

$$c^I(b\bar{b}) = \frac{3}{4}(1 - 3\rho_{00}) \left(1 - \frac{\alpha_s}{6\pi}(12 - 2.93)\right) = 0.708, \quad (57)$$

$$A_{FB}^I(b\bar{b}) = -\frac{3}{4}(\rho_{++} - \rho_{--}) \frac{2v_f a_f}{v_f^2 + a_f^2} \left(1 - \frac{\alpha_s}{6\pi}(6 - 1.37)\right) = 0.103, \quad (58)$$

$$\cos \theta \Big|_{\min}^I(b\bar{b}) = \frac{(\rho_{++} - \rho_{--})}{1 - 3\rho_{00}} \frac{2v_f a_f}{v_f^2 + a_f^2} \left(1 + \frac{\alpha_s}{\pi}(6 - 1.56)\right) = -0.145. \quad (59)$$

The numerical values obtained from the approximate formulas are quite close to the relevant numbers in Table 1 (column 4; tier 3) indicating that the approximation is quite good. The NLO finite mass effects in Eqs. (57–59) can be seen to reduce the respective leading NLO term by $\sim 25\%$. This is in accordance with the numbers in Table 1 and similar to what happens in the decay $W^+ \rightarrow c\bar{b}$ (system II).

The case $Z \rightarrow c\bar{c}$ has to be treated separately since, apart from the quark mass effects, one now has to use the electroweak coupling coefficients appropriate for up-type quarks. The numerical results are

$$\begin{aligned} H_U^{VV}(c\bar{c}) &= 4N_c q^2 \left[1 + \frac{\alpha_s}{6\pi}(2 + 0.367 + \dots)\right], \\ H_U^{AA}(c\bar{c}) &= 4N_c q^2 \left[0.999 + \frac{\alpha_s}{6\pi}(2 + 0.460 + \dots)\right], \\ H_F^{VA}(c\bar{c}) &= 4N_c q^2 \left[0.999 + \frac{\alpha_s}{6\pi}(0 + 0.599 + \dots)\right], \\ H_L^{VV}(c\bar{c}) &= 4N_c q^2 \left[0.001 + \frac{\alpha_s}{6\pi}(4 - 0.344 + \dots)\right], \\ H_L^{AA}(c\bar{c}) &= 4N_c q^2 \left[0 + \frac{\alpha_s}{6\pi}(4 - 0.307 + \dots)\right], \end{aligned} \quad (60)$$

or, using Eq. (47)

$$\begin{aligned}
H_{++}^I(c\bar{c}) &= 4N_c q^2 \left[0.189 + \frac{\alpha_s}{6\pi} (1.148 - 0.027 + \dots) \right], \\
H_{00}^I(c\bar{c}) &= 4N_c q^2 \left[0.000 + \frac{\alpha_s}{6\pi} (4.590 - 0.358 + \dots) \right], \\
H_{--}^I(c\bar{c}) &= 4N_c q^2 \left[0.957 + \frac{\alpha_s}{6\pi} (1.148 + 0.488 + \dots) \right].
\end{aligned} \tag{61}$$

For the unpolarized decay function H_{U+L} one obtains

$$H_{U+L}(c\bar{c}) = 4N_c q^2 \left[1.146 + \frac{\alpha_s}{6\pi} (6.886 + 0.103 + \dots) \right] \tag{62}$$

As expected, the mass corrections in the $(c\bar{c})$ case are smaller than those in the $(b\bar{b})$ case. According to (62) the $O(\mu_c)$ quark mass and the radiative corrections effects increase the zero mass Born term decay rate by 3.7%, i.e. one has

$$\Gamma(\text{NLO}; Z \rightarrow c\bar{c}, O(\mu_c)) = 1.037 \cdot \Gamma(\text{Born}; Z \rightarrow c\bar{c}, \mu_c = 0). \tag{63}$$

For the normalized angular decay distribution one finds

$$\widehat{W}^I(\theta)(c\bar{c}) = \frac{3}{8}(1 + \cos \theta)^2 \begin{Bmatrix} 0.551 \\ 0.537 \\ 0.538 \end{Bmatrix} + \frac{3}{8}(1 - \cos \theta)^2 \begin{Bmatrix} 0.449 \\ 0.439 \\ 0.440 \end{Bmatrix} + \frac{3}{4} \sin^2 \theta \begin{Bmatrix} 0.000 \\ 0.024 \\ 0.022 \end{Bmatrix}. \tag{64}$$

The coefficients of the distribution (64) are quite similar to those of the $(b\bar{b})$ case Eq. (56) only that the NLO finite mass effects are smaller than those in the $(b\bar{b})$ case. Apart from the small finite mass effects the shape of the angular decay distribution is also affected by the difference of the factor $2v_f a_f / (v_f^2 + a_f^2)$ in the $(b\bar{b})$ and $(c\bar{c})$ cases which are given by 0.94 and 0.67, respectively. This is relevant for the values of A_{FB} and c_f (see Eqs. (58) and (59)). As the relevant numbers in Table 1 show the $Z \rightarrow c\bar{c}$ forward-backward asymmetry is reduced by 28.9% relative to the $(b\bar{b})$ case. Also the minimum of the $c\bar{c}$ distribution is moved to the right by 28.4% going from the $(b\bar{b})$ to the $(c\bar{c})$ case (see Table 1).

7 Summary and Conclusions

We have presented $O(\alpha_s)$ results for the polarized decay functions that describe the decay of polarized (W, Z) bosons into massive quark–antiquark pairs. NLO quark mass corrections to the polarized decay functions have been found to be quite large. They can be as large as 200% of the leading NLO mass term. However, these large NLO quark mass effects do not feed through to the normalized angular decay distributions. The large NLO quark mass effects disappear when one is dividing out the total decay function in the normalized angular decay distribution.

We have combined these results with information on the spin density elements of the (W, Z) bosons in the two sample production processes $t \rightarrow b + W^+$ and $e^+e^- \rightarrow Z$ to write down explicit analytical and numerical forms of the polar angle decay distributions. The $O(\alpha_s)$ corrections to the polarized decay functions result in $O((1-3)\%)$ absolute changes in the angular coefficients of the polar angle decay distributions where the bulk of the NLO corrections come from the leading mass term. Quark mass corrections are small but, depending on the required accuracy, are non-negligible.

The radiative corrections make the angular decay distributions flatter for both W and Z decays where the main effect comes from the leading mass term in the final state NLO contribution. As a measure of the flatness of the decay distribution we have used the convexity parameter given by the second derivative of the decay distribution. For zero mass quarks the final state radiative corrections reduce the convexity parameters of the angular decay distributions by $\sim (6-8)\%$. Depending on the particular case under study, NLO quark mass effects reduce this value to $\sim (5-7)\%$. The forward-backward asymmetry is reduced by $\sim 1.5\%$ and $\sim 3.5\%$ for W and Z decays, respectively, where NLO finite mass effects have reduced these shifts by a small amount. The radiative corrections shift the position of the maximum or minimum of the decay distributions away from zero by $\sim 7\%$ and $\sim 3\%$ for W and Z decays, respectively, where again NLO finite mass effects

reduce these shifts by small amounts. The total width of the decay $W^+ \rightarrow c\bar{b}$ is increased by 3.8% by NLO effects where NLO nonzero quark mass effects account for 7.8% of the increase. The total widths of $Z \rightarrow b\bar{b}$ and $Z \rightarrow c\bar{c}$ are increased by 3.7%, where NLO mass effects account for 13% and 1.6% of the increase, respectively.

In this paper we have only discussed polar decay correlations which probe the diagonal spin density matrix elements of the production processes. If one wants to probe in addition the nondiagonal spin density matrix of the gauge bosons, one needs to involve in addition azimuthal correlations between the momenta of the particles involved in the production/decay process. Upon azimuthal averaging one would recover the results of the present paper (see e.g. [10]). A comprehensive discussion of the azimuthal correlations would form the subject of a separate publication.

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